Strategy Instruction and Error Prevention:

The Research behind Think*Fast!* for Basic Math Facts

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Think*Fast!*

FOR FLUENCY IN MATH

The design of ThinkFast is based on two primary considerations that have been established through research:

- Students learn the basic facts best when they understand the process and have efficient strategies for remembering the facts
- Students who have a conceptual understanding of the operation, and efficient strategies for remembering the facts, benefit from extended practice and are able to achieve fluency

With this in mind, there are two areas of relevant research that have provided the basis for the design of ThinkFast:

- Research on the relevance of strategies to successful development of fluency with basic facts
- Research on the design of effective and efficient practice for solving the basic facts

Because the number of relevant studies across these topics is far too large to address in this document, only an overview will be provided on the major issues.

Research on the relevance of strategies to successful development of fluency with basic facts

Many lines of research by many investigators over the past 75 years or so have converged to provide solid evidence that students use and benefit from the use of strategies in solving basic fact problems. The seminal work in this area was done by Brownell (Brownell and Chazal, 1935). Thiele (1938) provided the first experimental documentation that students who use strategies to solve basic facts are more successful in learning the facts than students who do not. Since that time, this position has been supported by a number of studies, including those of Swenson (1949) and more recent work by Thornton (1978; Thornton Jones and Toohey, 1983) as well as studies by Cook and Dossey (1983), Rathmell (1978), Steinberg (1985) and Woodward (2006).

These studies have been supported by more extensive lines of research into the question of how students typically develop fluency with the basic facts and how the use of strategies contributes to this development. This research has been conducted by many different researchers in math education, cognitive psychology and learning disabilities including Ashcraft (e.g., Ashcraft, 1983, 1985, 1992; Ashcraft and Battaglia, 1987; Ashcraft and Stazyk, 1981; Ashcraft and Fierman, 1982), Baroody (e.g., Baroody, 1983, 1985, 1989; Baroody, Ginsburg, and Waxman, 1983), Fuson (e.g., Fuson 1992a, 1992b), Geary (e.g., Geary, 1990, 1993, 1994, 1996), and Siegler (e.g., Siegler, 1986; 1987, 1988a, 1988b; Siegler and Shrager, 1984), among others.

> ***…the learning disabled student, left to his own resources, does not discover relationships and techniques that can help him remember the facts.***
>
> ***Thornton***
> *1978*

From these multiple lines of research, it is now well established that most students follow a similar course of development in learning math, and in the process, most students develop their own strategies for solving the basic fact problems (Cumming and Elkins, 1999). It is also known that over time, most students become more fluent with the basic facts, primarily by moving through a progression from less mature and efficient strategies to more mature and efficient strategies (Fuson, 1992; Geary 1994; Kilpatrick, Swafford, and Findell, 2001)

However, some caveats are very important. First, not all students are capable of deriving or developing their own strategies to solving the basic facts. (Cumming and Elkins, 1999; Geary, 1993; Rathmell, 1979) As Myers and Thornton (1978) noted,

> …the learning disabled student, left to his own resources, does not discover relationships and techniques that can help him remember the facts. Thus, if we just tell a child to memorize without more specific direction on "how to," we are not helping that child enough.

Second, it has been found that students with difficulty in math (as defined in Gersten Jordan and Flojo, 2005) move through the stages of maturity in solving the problems at a slower rate than students without such difficulties, and in the absence of intervention, may remain behind their peers indefinitely. (Geary, 1993; Goldman, Pellegrino and Mertz, 1988)

Third, for any student, it is important for them to understand the concepts of the addition, subtraction, multiplication and division operations prior to beginning practice with the basic facts (Brownell and Chazal, 1935). Once this conceptual level of understanding is reached, students without difficulty in math can benefit from repeated exposure to basic fact practice, because they are able to develop their own strategies, and progress from less mature and efficient strategies to more mature and efficient strategies over time. For these students, exposure to basic fact practice alone may be sufficient for them to progress to fluency. However, for students with difficulty in math, practice alone is not as effective, and in some cases, may never result in reaching fluency. (Hasselbring, Goin and Bransford, 1988; Howell, Sidorenko, and Jurica, 1987) A commonly reported observation with these students is a continued reliance on finger counting as a strategy for solving basic fact problems, although other inefficient strategies are also reported. (Geary, Bow-Thomas and Yao, 1992; Geary, Fan and Bow-Thomas 1992; Jordan, Hanich, and Kaplan, 2003)

For students with learning difficulties in math, the challenge is to provide the instruction and support they need to move from immature and inefficient strategies toward more efficient strategies, and ultimately, automatic retrieval, or fluency, with the basic facts. The first priority is to help them learn efficient and effective strategies that provide the necessary foundation for progress. The second priority is to provide an efficient and effective practice process that enables students to effectively make use of these strategies in developing fluency.

> ***"A new relationship is perceived by viewing numbers in terms of the important anchors, or benchmarks, of five and ten. Spatial relationships, including instant recognition of quantities in patterned sets, provide a global understanding of quantity as a special gestalt, or singular entity, going beyond counting."***
>
> *Van de Walle*
> *1990*

Strategy Instruction in ThinkFast

ThinkFast provides explicit instruction in the use of strategies to solve the basic facts. For addition and subtraction, it relies heavily on the composition and decomposition of numbers and their relationship to the anchors 5 and 10. These strategies have been used in studies by Thornton (1978), and Rathmell (1978), and advocated by Van de Walle (1990, 2003). As Baroody (1983) has noted, strategies such as these are very efficient in terms of cognitive processing, allowing the solution of many problems with a small set of procedural processes.

Fuson and her associates (e.g., Fuson, 1992; Fuson and Kwon, 1991a; Fuson and Kwon, 1991b; Fuson and Kwon 1992) have done extensive work on the use of these strategies in China, Japan and Korea. Hatano (1982) and Kroll and Yabe (1987) have also described the use of these strategies in Japan. Murata, Otani, Hattori, and Fuson, K. (2004) illustrate the teaching of these strategies in a Japanese classroom and note that they are part of the Japanese National Course of Study. The use of these strategies in these countries contributes to students becoming proficient in basic facts at a much earlier point than that of students in the U.S. (Fuson and Briars, 1990; Geary, Fan and Bow-Thomas, 1992; Song and Ginsburg, 1987) The vast majority of both Chinese and Korean students have been found to achieve fluency with basic addition and subtraction facts in the first grade (e.g., Fuson and Kwon 1992; Geary, 1996), whereas a comparable level of achievement is not reached by most US students until the third grade or later (Geary, 1996; Geary, Fan and Bow-Thomas, 1992).

Steinberg (1985) has documented that some U.S. students derive the strategies based on 10 as an anchor for themselves, but it is well documented that the English language makes it more difficult for students to discover these strategies, because of the irregular nature of the names for numbers between ten and twenty (Fuson and Kwon, 1991a, 1991b, 1992; Hatano, 1982; Yoshida and Kuriyama, 1991) However, Rathmell (1978) and Steinberg (1985) have found that U.S. students can apply these strategies effectively after being taught to use them.

In ThinkFast, strategies for multiplication and division are based on both the relationship of multiplication to addition and the patterns that emerge in the table of products of the digits 0 through 9. These strategies have been summarized in Kilpatrick Swafford and Findell (2001) For example, the student is taught that multiplying by 2 is the same as "doubling" the number, which the student has learned to do in developing fluency in addition. Multiplying by 9 is based on use of the fact that the product of any number and 9 is composed of two digits whose sum is 9, and the tens digit is one less than the number.

Research on how to design and schedule practice in solving the basic facts

The second innovative and unique characteristic of ThinkFast is the nature of the practice provided. It has long been generally accepted that practice is required for students to develop fluency with basic math facts (Ashlock, 1971; Brownell and Chazal, 1935; Geary, 1996; Rathmell, 1978). In fact, in the first part of the past century, instruction in basic facts was dominated by "drill" as the dominant form of practice, and drill constituted almost the entirety of instructional activity in teaching basic math facts. Brownell and Chazal (1935) are often cited as the first to issue a clear call for adding meaning to the process of teaching basic facts, but it is important to note that they did not advocate eliminating practice, just premature practice, before students understood the basic operations. The importance of such practice was later reiterated by Brownell (1956). More recently, others (e.g., Cumming and Elkins, 1999) have also noted that learning effective strategies for solving basic fact problems is not a guarantee that students will eventually become fluent with these facts. Additional practice is necessary as well, beyond the point that correct answers can be supplied.

> ***"Practicing incorrect answers is detrimental to increasing skills and in fact may impede the development of correct responses by strengthening the incorrect ones."***
>
> *Goldman and Pellegrino*
> *1987*

> ***"Specifically, the effect of arcade type drill and practice software on the automatization of addition facts had no effect on developing automaticity in mildly handicapped youngsters who used counting strategies."***
>
> ***Hasselbring, Goin, and Bransford***
> *1988*

A trivially obvious, but significant feature of traditional practice procedures is that they typically result in high levels of errors. Most practice procedures have been based on a similar format in which the student is presented a problem and asked to supply an answer. If the answer is correct, it is confirmed. If it is incorrect, some form of error correction is provided, which in most cases is simply providing the correct answer. Thus, the focus of the most common practice format is "error correction". Although an accepted feature of this "error correction" approach to practice is a relatively high percentage of errors, the occurrence of these errors is regarded as detrimental to the process of learning the facts, since it results in practice of incorrect answers. As Goldman and Pellegrino (1987) pointed out:

> Practicing incorrect answers is detrimental to increasing skills and in fact may impede the development of correct responses by strengthening the incorrect ones. Current information processing models of mathematics are quite explicit on this point (e.g., Siegler and Shrager, 1984; and see Pellegrino and Goldman, 1987, for further elaboration).

Geary (1993) made a similar observation:

> Computational accuracy is important because if the child commits many computational errors, then that child is more likely to retrieve incorrect answers during subsequent attempts to solve the problem (Geary, 1990; Siegler, 1986).

For many years, the nature of practice was accepted as a given, and little attention was devoted to the impact of the various features of the practice provided. For example, Pellegrino and Goldman (1987) noted that:

> What is less obvious is the form that such practice should take, from example, item set size, distribution of practice, and errorless training, as well as the amount of practice needed to produce

> rapid fact retrieval, particularly in children who are frequently characterized as slow learners (see, e.g., Hasselbring, Goin and Bransford, 1985).

About the time Pellegrino and Goldman made this observation, additional research began to be published on several new approaches to the organization and structure of practice that are relevant to this issue. A common thread throughout the research is the importance of reducing errors during practice as a means of more efficiently achieving both mastery of and fluency with basic math facts.

Carnine (1989) acknowledged the importance of reducing errors during practice and reviewed studies "which evaluated ways to reduce errors during acquisition (thus reducing acquisition time), in order to increase accuracy after the practice and foster automaticity. The studies deal mostly with increasing accuracy and decreasing instructional time to mastery, both of which set the stage for automaticity." Carnine (1989, p. 603)

More recently, a number of other practice techniques have been formalized and evaluated through research that have been found to provide significant benefits when applied to the task of promoting fluency with basic math facts. These procedures all share the ability to reduce the number or errors students make while learning correct answers to basic fact problems and developing fluency in answering. These procedures represent a departure from traditional approaches to practice in that they replace the "error correction" format with procedures that can be described as an "error prevention" format. The focus is on structuring practice so that the student acquires the ability to supply the answer to a problem with a very low incidence of errors.

> ***"Due to the history of failure that the majority of special education students have experienced, procedures should be developed that can limit the number of errors committed."***
>
> ***Cybriwsky and Schuster***
> *1990*

Among these procedures are Cover, Copy and Compare (Skinner, Bamberg, Smith and Powell, 1993; Skinner, McLaughlin, and Logan, 1997; Skinner, Turco, Beatty, Rasavage, 1989), stimulus fading, and response prompting procedures (Wolery, Ault and Doyle, 1992). In comparisons of these procedures, one of them found to be most effective is a response prompting procedure - a "delayed prompt" procedure (Poncy, Skinner and Jaspers 2007). The delayed prompt procedure uses either of two types of delay, a "constant time delay" or a "progressive time delay". Reviews of research on the delayed prompt procedure with constant time delay have concluded that it is effective in a wide range of studies by different investigators across many content areas (Handen and Zane 1987; Stevens and Schuster 1988; Wolery, Holcombe, Cybriwsky, Doyle, Schuster, Ault and Gast 1992). Several studies have used the procedure for teaching basic math facts (Cybriwsky and Schuster, 1990; Koscinski and Gast, 1993a; Koscinski and Gast, 1993b; Koscinski and Hoy, 1993; Mattingly and Bott, 1990; Morton and Flynt, 1997; Williams and Collins, 1994). These studies consistently show that the process results in very low error rates and rapid mastery of and fluency with the basic facts.

> ***"The time delay procedure minimizes student errors because the prompt is consistently provided when the student does not respond independently."***
>
> ***Stevens and Schuster***
> *1988*

The delayed prompt procedure with constant time delay is the basis of the practice procedure used in ThinkFast. The most significant property of the delayed prompt procedure is that it greatly reduces the number or errors students make during practice in answering basic fact problems. The frequency of errors among students learning basic math facts varies with grade level and subgroup, but students with learning disabilities characteristically exhibit a greater frequency of errors for basic fact problems than students without disabilities and gifted students. Error rates of 20% to 50% are not uncommon in assessment of students who have difficulty learning basic addition and subtraction facts (e.g., Geary, 1990; Geary Bow-Thomas and Yao 1992; Geary, Brown and Samaranayake 1991) and even higher for multiplication and division facts (Van Luit and Naglieri, 1999). With the delayed prompt procedure, error rates are typically less than 5% for this same category of students. For that reason, the delayed prompt practice procedure is often referred to as a "nearly-errorless" procedure.

How does the delayed prompt procedure prevent errors and reduce their frequency? The answer has two important components. First, a primary feature of the procedure is that when students are unsure of an answer, they are provided a "prompt" that helps them remember the correct answer. It's called a "delayed" prompt because the prompt is provided after a specified delay if the student has not yet answered the problem. In effect, this means the student is provided help only if they need it – and evidence of their need for help is their delay in answering.

Second, the nature of the prompt used in ThinkFast is very important. With this procedure, the prompt can be any information that will insure that the student can provide the correct answer. In many situations, the prompt is simply having the teacher say the correct answer, which the student must then write as the answer to the problem written on paper. In others (e.g., Williams and Collins, 1994), the prompt is provided in the form of manipulatives the student can use to help determine the answer.

> ***"Results indicated that the near-errorless learning procedure was effective and efficient in teaching multiplication facts to students with learning disabilities. Learning generalized to a paper-and-pencil task, to a different presentation orientation, and to the reverse fact."***
>
> ***Koscinski and Gast***
> *1993a*

In ThinkFast the prompt provided to the student is a summary of the strategy for solving the problem that was taught earlier in the program. Thus, during practice, students are given a short "refresher" in how to apply the appropriate strategy. What this means, in effect, is that the focus of practice is shifted from just rote "pairing" of problem and answer in an "associative" manner (Hasselbring, Goin and Bransford, 1988) to an emphasis on having students practice mathematical thinking that leads to the answer. This is the essence of the use of strategies to facilitate the development of both accuracy and fluency in learning the basic facts. With the "error-prevention" format, students practice using the strategies repeatedly, and the use of the strategy prevents errors and leads to fluent performance.

Finally, Rathmell (1978) has noted that "Situations that call for immediate response are very frustrating to children who have no thought processes available that permit them to answer in the time allotted." One of the benefits of the delayed prompt procedure is that it takes the frustration out of the process. It provides help whenever the student is unsure of an answer and he/she is never pressured to come up with an answer in the absence of a strategy for deriving it.

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